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ABSTRACT

Researchers and educators are calling for increased use of technology and attention to function concepts in school mathematics. Students often have considerable difficulty gleaning pointwise and global information from Cartesian (R squared) representations of functions, whether they are hand- or machine-produced. Described here is an interactive computer-based learning environment (the Function Explorer) which provides dynamic, linked representations of functions. Representations using tables, parallel number lines, and perpendicular number lines dynamically display ordered pairs of the function. A randomized comparative experiment is described which was performed to test the effectiveness of the number line representations for enhancing student understanding of basic function concepts. Contains 13 references. (Author/MKR)

The Effect of the Use of Number Lines Representations on Student Understanding of Basic Function Concepts

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Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

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THE EFFECT OF THE USE OF NUMBER LINES REPRESENTATIONS ON STUDENT UNDERSTANDING OF BASIC FUNCTION CONCEPTS

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Researchers and educators are calling for increased use of technology and attention to function concepts in school mathematics. Students often have considerable difficulty gleaning pointwise and global information from Cartesian (R^2) representations of functions, whether they are hand- or machine-produced. Described here is an interactive computer-based learning environment (the Function Explorer) which provides dynamic, linked representations of functions. Table, parallel number lines, and perpendicular number lines representations dynamically display ordered pairs of the function. A randomized comparative experiment is described which was performed to test the effectiveness of the number lines representations for enhancing student understanding of basic function concepts.

The function concept is unifying and central to the understanding of mathematics and its applications. Research from the last two decades has detailed numerous and deep misconceptions and difficulties students have with the function concept (see Leinhardt, Zaslavsky, & Stein, 1990). This study was primarily concerned with the difficulties students have interpreting graphs. While students can often produce graphs from equations and tables, the reverse process of obtaining pointwise, and especially global, information from graphs is difficult for students (see Kieran, 1992). Educators and researchers see the use of computers as beneficial for the teaching of functional thinking (see Dubinsky & Tall, 1991).

Researchers raise two concerns which hinder the use of technology for teaching functional thinking. First, Kaput (1992) warns that the "retrofitting" of general application (expert) tools as tools for learning mathematics is not easy and may not be effective. Goldenberg (1991) and other researchers (see Moschkovich, Schoenfeld, & Arcavi, 1993) have found that students are not always gleaning the correct information, or insights, from the ("perfect") graphs presented by graphing software and graphing calculators. In fact, novices may pick up on the wrong aspects of what they see—what they see being effected by what they know. Secondly, the (static) Cartesian graph itself is particularly difficult for novices to interpret. Goldenberg, Lewis, and O'Keefe (1992) see "that the act of representing functions graphically [in R^2] has as much potential to produce confusion as enlightenment" (p. 240).

This researcher has developed an interactive computer program incorporating perpendicular number lines *and* parallel number lines to represent functions. The design of the representations, user-interface (the program is titled the *Function Explorer*), and learning activities used in this study are based on psychological, mathematical, and historical considerations. Piaget, Grize, Szeminska, and Bang (1977) found that the root of the function concept, *pairing*, is present in the minds of children at the preoperatory level. This elementary form of cognitive structuring allows the child to conceive of an action of starting with an object and determining a corresponding object (for example: child, mother; sheep, shepherd).

The primary purpose of the *Function Explorer* is, upon input by the user, to display a single ordered pair of the function. The *Function Explorer* has three representations where input of the value of the independent variable is possible: a table, parallel number lines, and perpendicular number lines. Input in the number lines representations is accomplished by moving the mouse pointer to the location on the input ("x") number line. Input via the table is accomplished by mouse-clicking the table and typing the value desired. For each new value of the independent variable, the corresponding function value is (instantaneously) updated. The input-output pair appears simultaneously in all three representations. In the case of the parallel number lines, the output ("y") is displayed on the output number line. The parallel number lines elaborate the ordered pair notion, providing the student a bridge between the table—which provides a *pair of numbers*—and the Cartesian graph (perpendicular number lines)—which provides a *single geometric point.* The parallel number lines are situated horizontally because this is the orientation students are familiar with in middle school and algebra textbooks.

The representations of the *Function Explorer* are dynamic. Interest in properties of moving bodies helped spurn development of the calculus in the 17th-century. Over time however, the *direction of motion* of a moving body at a point has evolved to be the *tangent to the curve* (Kline, 1972). Using the perpendicular number lines representation the student may witness and consider a point *moving* in the plane. (Computer speed of at least 33MHz is required for the three linked representations to update "simultaneously.")

Discrete points may be graphed and entered in the table permanently, by clicking the mouse button. However, the program does not produce a complete graph. The *Function Explorer* is designed to be a learning environment, by providing a scientific instrument for investigating functions—which is more like a microscope and less like a VCR. The program is intended to *aid understanding* and *not* designed to *produce human products* (i.e., complete $\_^2$ graphs).

Parallel number line representation of functions has been investigated by Friedlander, Rosen, and Bruckheimer (1982) and Arcavi and Nachmias (1993). In both these cases input and output values on the respective number lines are connected with lines, and multiple pairs are shown at the same time. The purpose of this representation was not to make connections to a table and Cartesian graph and the representation is not dynamic.

A parallel number line representation similar to that of the *Function Explorer* (input value on the top number line can be manipulated dynamically, and connection lines not used) is described in Goldenberg, Lewis, and O'Keefe's (1992) article and in O'Keefe's (1992) doctoral dissertation. O'Keefe found that the dynamic parallel number lines environment (*DynaGraph* program) accurately conveyed important features of functions, including dependence, relative change, and critical values. *DynaGraph* did not display a Cartesian graph.

Methodology and Data Sources[1]

To test the instructional effectiveness of the number lines representations, a randomized comparative experiment was performed. Four eighth-grade classes (n=74), all taught by one teacher, were used for this study (two Pre-Algebra and two Algebra I classes). Students in each class were randomly assigned to two treatment groups. Students in both treatment groups were in the same classroom, at the same time. To vary the treatment, each student was given a login name. When a student logged on to the computer, the correct version of the program was automatically loaded (depending on the assigned treatment group). The first treatment group (PNL group) used the *Function Explorer*, with all its representations displayed (table, parallel number lines, and perpendicular number lines). The second group (No PNL group) used a version of the program, which had the parallel number lines representation hidden (table and perpendicular number lines shown). All students used their version of the program to solve problems on worksheets—the content of which was taken from the curriculum in use. Data was gathered from a pre- and post-questionnaire on functions, an opinion survey, taped student interviews, and audit trails of learner interaction with the software (a design feature which keeps track of what representations the user is accessing, and when). A repeated-measures ANOVA ($p = .05$) was used to analyze the questionnaire results.

On day one of the experiment the pre-questionnaire was administered. On day two, a 5-10 minute introduction/demonstration of the *Function Explorer* was given. For the remainder of days two through six, students solved worksheet problems using the *Function Explorer.* On day seven, the post-questionnaire was administered. The questionnaire had five subtests. Content of the questionnaire was determined before the worksheet content and subjects were selected. All graphs on the questionnaire were Cartesian graphs.

Results

Questionnaire results showed that both groups showed significant improvement on the subtests involving pointwise interpretation and global interpretation of graphs. There was no significant change on subtests on the definition of function, use of letters in function notation to stand for varying quantities, and the relationship between the formula and the graph. There was no significant difference between the treatment groups, and no time-group interaction was found.

The audit trail data reports the number of uses and length of time a student used the three representations (table, parallel number lines, and perpendicular number lines). Since the parallel number lines representation had never been seen by the students previous to the study, it was unclear if the students would even use the representation. All the representations were used from the outset, and the five-day

[1] This study is a dissertation study completed by the author at the University of Northern Colorado (Ph.D. in Educational Mathematics). Learning activities, function questionnaire, and additional numeric data from this experiment are available from the author.

averages showed students in the PNL group used the parallel number lines representation 26% of the time, the perpendicular number lines 17% of the time, and the table 10% of the time. The five-day averages in the No PNL group showed the students using the perpendicular number lines 46% of the time and the table 10% of the time. Students reported in the surveys and interviews that they preferred the number line representations over the table because it was quicker (using the mouse) than typing in numbers. However, they *did use* the table for output. That is, they would mouse-point to an input value, and watch the table for the input-output pair.

Audit trail, interview, and survey results establish that the use of the representations varied significantly between students. Some students who had the parallel number lines, preferred rather to use the perpendicular number lines. On the other hand one student said she rarely used the perpendicular number lines and went so far as to say, "Take it out," because it was confusing. Some students felt the parallel number lines representation was clearer than the perpendicular numbr lines because the values of x and y were separate and more readable. The students did feel that the program helped them understand functions and graphs, and that the program was easy to use. The students liked the individual control the program offered. Those who compared the program to a graphics calculator, preferred the program because it was "easier to get exact answers," and the user control afforded by the mouse.

Discussion

This study has shown that interpretation of graphs can be taught successfully with number lines representations. One of the things that makes the global interpretation tasks difficult for students is that they must think about, and report sets of numbers and intervals of numbers, rather than one discrete number. The ability to use the mouse pointer to move back and forth within an interval to select (and input) values, helped students think about intervals of numbers.

No improvement was shown on the questionnaire subtests on the definition of function, use of letters in function notation to stand for varying quantities, and the relationship between the formula and the graph. First, these concepts were not taught during the study. Secondly, these notions are abstractions, and "just being in the presence" does not cause the student to construct these generalizations.

Future research is needed to better determine the appropriate use of multiple representations of functions. Many of the students in this study could answer the worksheet questions in the presence of the dynamic representations, but could not answer the analog questions reading a (static) Cartesian graph. Follow-up "off-line" learning activities may enhance the on-line activities. These representations need to be tested at other levels and with other concepts. These dynamic representations may be used to teach concepts from integer arithmetic to the calculus concepts of change.

The parallel number lines representation may give students who are not yet able to interpret Cartesian graphs access to function concepts. This may become increasingly important as school algebra is reorganized around the concepts of functions, families of functions, and mathematical modeling (see Heid, 1995). The *Function Explorer* is not designed to replace other graphing technologies, but is more as a forerunner, preparing students to more effectively interpret machine-produced graphs.

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